The position of a sphere of mass 60-g oscillating at the end of a spring is given by x (t) = (3.0 cm)[cos(8t)] , where t is in seconds. What is:

a) The amplitude
b) The time period
c) The spring constant
d) The maximum speed
e) The total energy
f) The velocity at t = 0.50-s

Given:

Mass of the sphere: m = 60 g = 0.06 kg

Equation of the simple harmonic motion: x (t) = (3.0 cm)[cos(8t)]

Determine:

a) Amplitude of oscillation: A

From the equation of SHM:

A = 3.0 cm = 0.03 m

b) The time period of oscillation: T

From the equation of SHM:

Angular velocity: $\omega = 2\pi / T = 8$ rad / s

$$\mathbf{T = 2\pi / 8 = 0.78\ s}$$

c) The spring constant: k

Use equation:

$$T = 2\pi \times (m / k)^{1/2} \text{ ------------------(1)}$$

Squaring both sides of (1), rearranging (1) & substituting for T and m in (1):

$$\mathbf{k = 4\pi^2 m / T^2 = [\, 4 \times (3.14)^2 \times 0.06 \,] \,/\, (0.78)^2 = 3.9\ N/m}$$

d) The maximum speed of the oscillation: v_{max}

Use formula:

$$v_{max} = \omega A \text{ ---------------------------(2)}$$

Substituting for ω & A in (2):

$$\mathbf{v_{max} = 8 \times 0.03 = 0.24 \text{ m / s}}$$

e) Total energy: E

Use equation:

$$E = (½)kA^2 \text{ --------------------------------(3)}$$

Substituting for k, & A in (3):

$$\mathbf{E = (½) \times 3.9 \times (0.03)^2 = 1.8 \times 10^{-3} \text{ J}}$$

f) Velocity at t =0.50-s : v

Velocity can be calculated from the first derivative of the position equation of SHM with respect to time.

$$v = dx / dt = d[(3.0 \text{ cm}) \cos(8t)] / dt = (3.0 \text{ cm} \times 8) \times [- \sin(8t)] \text{ --------(4)}$$

Substituting for t in (4):

$$\mathbf{v = 24 \times [- \sin (8 \times 0.50)] = 18 \text{ cm/s} = 0.18 \text{ m/s}}$$