A ball of mass 0.5 kg is rotated in a vertical circle at the end of a string 75 cm in length. The frequency of rotation is 2 rev/sec.

a) What is the tension at the bottom of the loop?
b) What is the tension at the top of the loop?

Given:

Mass of the ball	$m = 0.5$ kg
Radius of the circular path	$r = 75$ cm $= 0.75$ m
Frequency of rotation	$f = 2$ rev /sec
Acceleration due to gravity	$g = -9.8$ m/s^2

a) Tension at bottom of loop

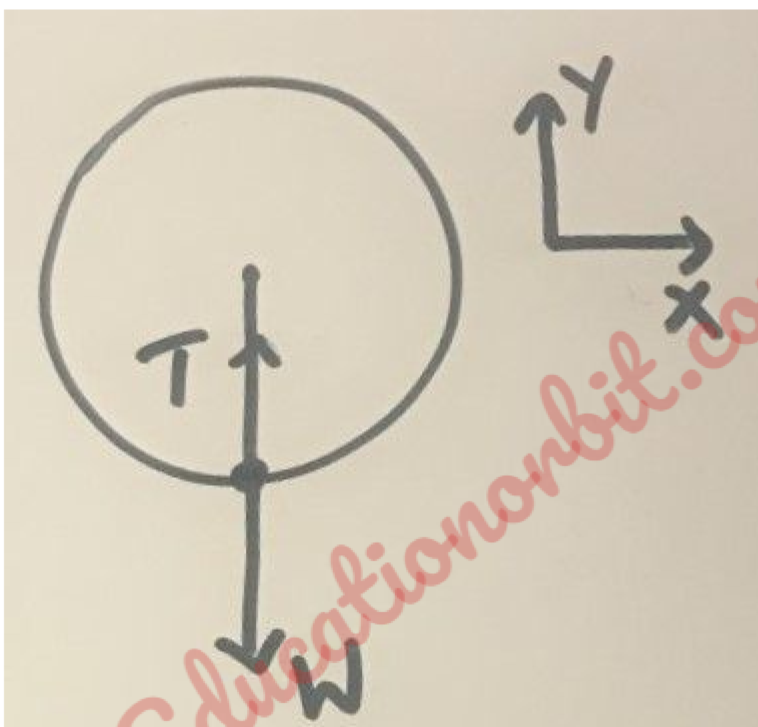

Fig (1)

At the bottom of the vertical loop, the tension T on the string provides both the force necessary to overcome the weight of the ball acting in the downward direction and the centripetal force required to keep the object in the circular path. Centripetal force is "mv^2 / r" and acts towards the center of the circular path. It is in the upward direction in this case.

From Fig (1), the net force at the bottom of the circular path is:

$$mv^2 / r = T - W = T - mg \text{ -----------------(1)}$$

Rearranging (1):

$$T = (mv^2 / r) + mg \text{ ---------------------(2)}$$

But $v = \omega r = 2\pi fr$

Then (2) becomes:

$$T = m \times (4\pi^2 f^2 r + g)$$

$$\mathbf{T = 0.5 \times [(4 \times 3.14^2 \times 2^2 \times 0.75) + | - 9.8 |] = 64 \text{ N}}$$

b) Tension at the top of the loop

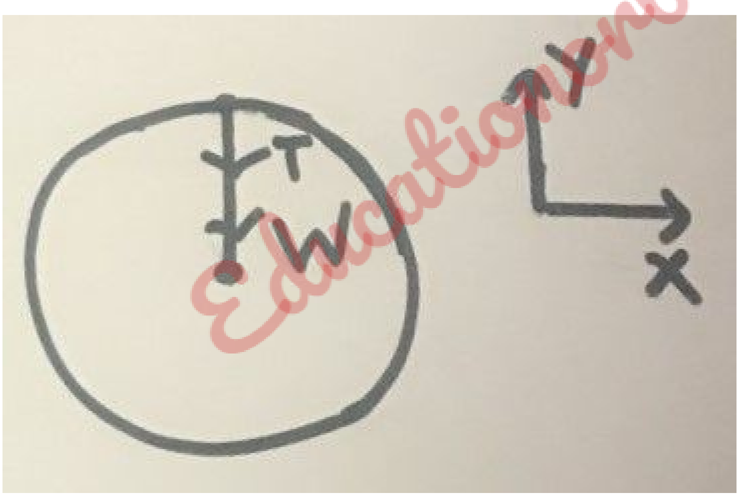

Fig (2)

At the top of the vertical loop, both the tension T on the string and the weight of the ball acting in the downward direction provide the centripetal force required to keep the object in the circular path. Centripetal force is "mv^2 / r" and acts towards the center of the circular path. It is in the downward direction in this case.

From Fig (2), the net force at the top of the circular path is:

$$mv^2 / r = T + W = T + mg \text{ ----------------(3)}$$

Rearranging (3):

$$T = (mv^2 / r) - mg \text{ -------------------------(4)}$$

But $v = \omega r = 2\pi f r$

Then (4) becomes:

$$T = m \times (4\pi^2 f^2 r - g)$$

$$\mathbf{T = 0.5 \times (4 \times 3.14^2 \times 2^2 \times 0.75 - | - 9.8 |) = 54\ N}$$